Math317
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First Exam

(1) A group of six software packages available to solve a linear programming problem has been ranked from 1 to 6 (best to worst). An engineering firm, unaware of the rankings, randomly selected and then purchased two of the packages. Let S denote the set of all possible outcomes for the firm's selection. Let A denote the subset of outcomes corresponding to the selection of at least one of the two best. Let B denote the subset of outcomes corresponding to the selection of the best and one of the two worst. List the outcomes in S, A and B. Find $P(A)$ and $P(B)$.

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(2) Professor Stern has three cars. The probability that on a given day car 1 is operative is 0.95, that car 2 is operative is 0.97, and that car 3 is operative is 0.85. If Professor Stern's cars operate independently, find the probability that next Thanksgiving day

(a) all three of his cars are operative;

(b) at least one of his cars is operative;

(c) only one of his cars is operative.

$^{135}P_4 + {}^{134}P_4$

(3)(a) There are 12 students in a class. What is the probability that their birthdays fall in 12 different months? Assume that all months have the same probability of including the birthday of a randomly selected person.

(b) A club of 136 members is in the process of choosing a president, a vice president, a secretary, and a treasurer. If two of the members are not on speaking terms and do not serve together, in how many ways can these four people be chosen.

(c) In how many ways can 23 identical refrigerators be allocated among four stores so that one store gets eight refrigerators, another four, a third store five, and the last one six refrigerators? If there are 4 defective refrigerators among the lot, what is the probability that each store will receive one defective refrigerator?

$P(\overline{CA} \wedge C) = P(\overline{CA}/C)P(C) = 0.016$

(4) Roads A, B, and C are the only escape routes from a state prison. Prison records show that, of the prisoners who tried to escape, 30% used road A, 50% used road B, and 20% used road C. These records also show that 80% of those who tried to escape via A, 75% of those who try to escape via B, and 92% of those who try to escape via C were captured. What is the probability that a prisoner who succeeded in escaping used road C?

$P(A) = 0.3$
$P(B) = 0.5$
$P(C) = 0.2$

$P(\overline{CA})$
$P(C/\overline{CA}) = 0.9796$

$P(\overline{CA}/C) = 1 - P(CA/C) = 0.08$

$P(CA/A) = 0.8$
$P(CA/B) = 0.75$
$P(CA/C) = 0.92$

(5) A newly married couple decides to continue having children until they have one of each sex, but they agree that they will not have more than four children even if all are of the same sex. The events of having a boy and a girl are independent and equiprobable. Let the random variable Y represent the number of children the couple will have.

$P(C/\overline{CA}) = \dfrac{P(C \wedge \overline{CA})}{P(\overline{CA})}$

Please turn over

$P(CA/A) = \dfrac{P(CA \wedge A)}{P(A)}$

$P(C \wedge \overline{CA})$

$P(\overline{CA}) = 0.204$

$P(CA) = P(CA \wedge A) + P(CA \wedge B) + P(CA \wedge C) = 0.799$

(a) List all possible outcomes in the sample space;
(b) Find the probability distribution for Y;
(c) How many children should this couple expect?
(d) Find the standard deviation of the number of children the couple will have.

(6) The atoms of a radioactive element are randomly disintegrating. If every gram of this element, on average, emits 3.9 alpha particles per second, what is the probability that during the next second the number of alpha particles emitted from 1 gram is

$\lambda = 3.9$

(a) at most 3;
(b) at least 2;

$P(Y \leq 3)$ $P(Y \geq 2)$

(c) Derive the moment generating function for the Poisson random variable and use it to find the mean and the variance;
(d) Is it likely that during the next second the number of alpha particles emitted from 1 gram is 10? Explain.

(7) Sherry and Ann play a series of backgammon games until one of them wins five games. Suppose that the games are independent and the probability that Sherry wins a game is 0.58. Find the probability that the series ends in seven games and Sherry wins.

(8) Florence is moving and wishes to sell her package of 100 computer diskettes. Unknown to her, 10 of those diskettes are defective. Sally will purchase them if a random sample of 10 contains no more than one defective disk. What is the probability that she buys them?

$P(S) = 0.58$

$P(Y=7) = {}^{6}C_{4}\,(0.58)^5(0.42)^2$

$$\frac{{}^{10}C_0 \times {}^{90}C_{10} + {}^{10}C_1\,{}^{90}C_9}{{}^{100}C_{10}}$$

Good Luck